

# Composition, Electrical Conductivity, and Total Radiation of Nitrogen Plasma

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## Theme

**M**ETHODS to calculate the composition, the electrical conductivity and the total radiative source strength of nitrogen plasma at temperatures up to 30,000 K are presented. It is supposed that the plasma is in the state of local thermodynamic equilibrium. At a pressure of 1 atm, the electrical conductivity and the continuum radiated power, calculated in the present paper, are essentially in better agreement with experimental data than other known theoretical results. A semi-empirical equation is presented which makes it possible to take the line radiation into account in calculating the total radiative source strength. This equation makes possible the extrapolation of the line radiation with respect to pressure and temperature.

## Contents

In nitrogen plasmas up to 30,000 K, the following reactions have to be considered: dissociation, first ionization, and second ionization. These reactions are coupled. By application of the law of mass action on the above reactions, we obtain three equations for the determination of the mole fractions of  $N_2$ ,  $N$ ,  $N^+$ ,  $N^{++}$ , and  $e^-$ . Additionally we get two equations from Dalton's law and from the condition of quasineutrality.

The simultaneous solution of the complete system, consisting of three nonlinear and two linear equations, is complicated and time-consuming. Similar to a proposition of Frie (see also Artmann and Bohn<sup>1</sup>) in this paper the basic components  $N^{++}$  and  $e^-$  are introduced. All particles, which appear in the nitrogen plasma, contain these basic components. In this way it is possible to reduce the set of equations to two nonlinear equations which can be solved iteratively using Newton's method.

Special care was taken for the calculation of the partition functions appearing in the law of mass action. In the electronic ground state the internal partition function of the  $N_2$ -molecule is derived from the model of the anharmonical vibrating rotator. This leads to a semiconvergent series. It is truncated at the vibrational quantum number that yields a minimum of the general term of the series. Then the remaining term belonging to the partial sum reaches its smallest value and cannot be minimized. In the cases of higher electronic excitation, the internal partition function of the molecule is calculated according to the model of the harmonic oscillator and rigid rotator.

The internal partition functions of the atom and of the ions are calculated by summation over all energy levels up to the ionization energy, reduced by the coulomb interaction of the particles.

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Figure 1 shows the plasma components, found in the described manner. They deviate strongly from former results of Burhorn and Wienecke.<sup>2</sup> These deviations increase with increasing pressure. The reasons for the observed discrepancies were discussed extensively in an earlier paper.<sup>7</sup> The main reason is to find how to calculate the internal partition functions in the different ways. In previous papers<sup>2</sup> the partition function is set equal to the statistical weight of either the basic state ( $N$ ) or the basic multiplett ( $N^{++}$ ), or the summation is only extended over two or three subsequent energy levels ( $N^+$ ). For this reason the partition functions in previous papers are smaller than in the present one. The pressure dependence of the deviations originates in the growing reduction of the ionization energy in case of increasing pressure. With the presently calculated plasma composition, the electrical conductivity and the radiative source strength of the nitrogen plasma are calculated.

The calculation of the electrical conductivity is based on the first Chapman-Enskog approximation. In this paper the complete kinetic-theory formula for the electrical conductivity is not used; instead, this formula is approximated in the same manner as done by Yos.<sup>3</sup> This approximation gives an explicit equation for the electrical conductivity in terms of the collision integrals, which describe the interaction between the electrons and the heavy particles.

Take, for example, a fully ionized gas, consisting of electrons and ions of different charges. The calculation of conductivity requires the collision integrals of the Coulomb collisions  $e^- - N^+$ ,  $e^- - N^{++}$ , etc. In the present paper these collision integrals are estimated in such a manner that the calculated electrical conductivity of the regarded gas will always be in agreement with the generalized theory of Spitzer and Härm.<sup>4</sup>

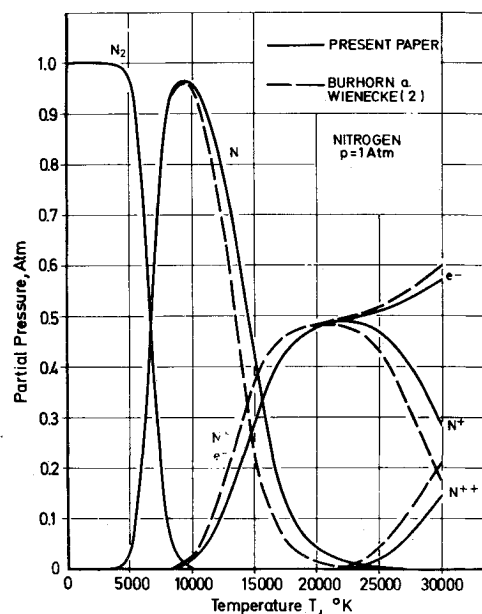


Fig. 1 Composition of nitrogen plasma.

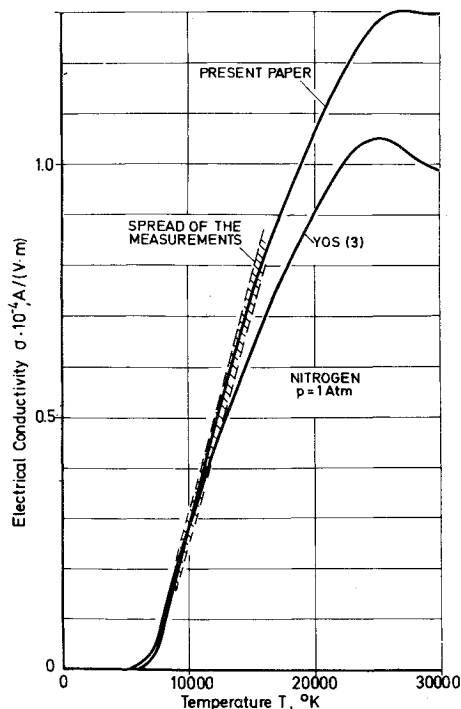


Fig. 2 Nitrogen electrical conductivity.

To calculate the electrical conductivity of a partially ionized gas, in the temperature range up to 30,000 K, we need, in addition, the collision integrals for the collisions of the electrons with the neutral particles. For the collisions  $e^- - N_2$  the collision integrals are calculated using the cross sections measured by Crompton and Sutton.<sup>5</sup> For the collisions  $e^- - N$  the collision integrals in the present paper are roughly approximated by the half of the collision integrals of the collisions  $e^- - N_2$ . Figure 2 shows the calculated electrical conductivity of a nitrogen plasma at atmospheric pressure. Between 9000 and 16,000 K the calculated curve always remains within the scatter range of the data, measured by different authors. The curve calculated by Yos,<sup>3</sup> deviates from this region already at 13,000 K and yields at 30,000 K values, which are 30% lower than those calculated in the present paper.

The continuum radiated power is in the present paper calculated in accordance with the semiclassical theory of Kramers.<sup>6</sup> The same formula has been used by Yos.<sup>3</sup> Figure 3 shows the results of this calculation. The figure also presents the spread of total radiative source strength measurements performed by different authors. The present calculated continuum radiated power lies everywhere below this spread. This is probable because our calculations, at present, do not take into consideration the line radiation. But line radiation is included in the measurements of the total radiative source strength. The continuum values calculated by Yos show a steeper tendency, and enter at 9500 K into the spread of the total radiative source strength measurements. The differences between the results presented here and the results of Yos may have been caused only by the different plasma compositions used. The better agreement of the presently

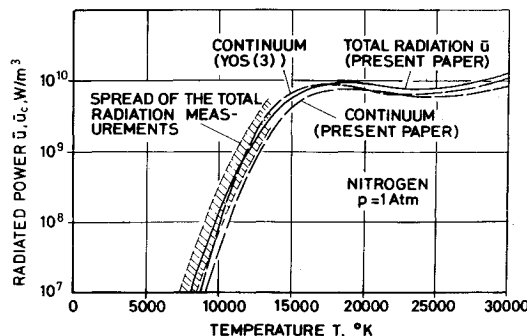


Fig. 3 Nitrogen continuum radiated power, and total radiative source strength.

calculated continuum radiated power with the measurements, therefore, is a good indirect confirmation of the plasma composition calculated in the present paper.

To estimate the contribution of the line radiation to the total radiative source strength, a semiempirical equation is used. This equation contains a set of constants which can be estimated either empirically or by approximation of the result of a non-repetitive calculation. If these constants are known, it is possible to extrapolate the line radiation with respect to pressure and temperature. In Fig. 3 the calculated total radiative source strength, composed of the continuum and the line radiation, is plotted too. This curve lies between 8000 K and 13,000 K within the spread of the total radiative source strength measurements.

It may be expected that with the plasma composition, calculated in the present paper, and the derived collision integrals an improved calculation of other thermodynamic and transport properties of nitrogen is also possible.

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